On Time Series Modeling of Nigeria's External Reserves

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This paper proposes three short-term forecasting models for the adjusted external reserves using the seasonal autoregressive integrated moving average (SARIMA), seasonal autoregressive integrated moving average with an exogenous input (SARIMA-X) and an autoregressive distributed lag (ARDL) processes. The performances of the proposed models are compared with the existing model obtained using an autoregressive integrated moving average (ARIMA) process using the pseudo-out-of-sample forecasting procedure over July 2013 to May 2014. The results show that SARIMA model outperformed the other models in three to six months forecast horizon, whereas ARDL model performs better in one to two months forecast horizon. Therefore, in forecasting external reserves in longer horizon, the paper concludes that seasonality should be accounted for by using the SARIMA model.

Keywords: External Reserves, ARIMA, SARIMA, SARIMA-X, ARDL, Statistical Loss Functions

JEL Classification: E17, E31

1.0 Introduction

The IMF (2009) defines external reserves as those external assets that are readily available to and controlled by monetary authorities for meeting balance of payments financing needs, for intervention in foreign exchange markets to affect the currency exchange rate, for other related purposes such as maintaining confidence in the currency and the economy, and serving as a basis for foreign borrowing. It also defines external reserves as the currency deposits of Central Banks used in meeting the objectives of safeguarding currency stability. In his opening remarks at an IMF/World Bank International Reserves – Policy Issues Forum, Fischer (2001) writes:

> *Reserves matter because they are the key determinant of a country's ability to avoid economic and financial crisis. This is true of all countries, but especially of emerging markets open to volatile international capital flows…………. The availability of capital flows to offset current account shocks*

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> *should, on the face of it, reduce the amount of reserves a country needs. But access to private capital is often uncertain, and inflows are subject to rapid reversals, as we have seen all too often in recent years. We have also seen in the recent crises that countries that had big reserves, by and large, did better in withstanding contagion than those with smaller reserves – to an extent that is hard to account for through our usual analyses of the need for reserves.*

The management of external reserves of a country is the exclusive responsibility of the country's Central Bank. In Nigeria, the Central Bank of Nigeria (CBN) Act 1991 vests the custody and management of the country's external reserves in the CBN. Before financial globalization, reserves were held by countries mainly to manage foreign exchange demand and supply arising from current account transactions. The traditional rule of thumb for Central Banks was that they should hold a quantity of foreign exchange reserves equivalent to three months of imports, but during the great depression of the 1930's, Keynes advocated the use of foreign reserves for mitigating external vulnerability or shocks. He called for an international clearing system where the main source of liquidity would be related to the value of trade.

The level of external reserves has remained an important parameter in gauging the ability of economies to absorb external shocks. This is based on the belief that a country's own reserve holding is a critical component of her insurance against possible external shocks. While there is little consensus on what constitutes an adequate level of reserves for a country, some traditional metrics such as trade based measure, money supply based measure, foreign exchange disbursements measure and financial account measure have been developed to guide countries in their reserves holding decisions. The trade based measure gives an indication of the trade financing capacity of the external reserves by yielding the number of months of import the reserves level can finance. International best practice requires that a healthy country's level of external reserves should be adequate to finance three to six months of a country's merchandise imports.

The money supply based measure is computed as the ratio of external reserves to money supply, with the standard benchmark being set at between 5 and 20 per cent. The foreign exchange disbursement measure is crucial in countries where the central bank is the dominant supplier of foreign exchange needs. Also, in view of the importance of foreign portfolio investment to emerging markets, an assessment of the ratio of external reserves to portfolio investment is of significant policy relevance. It is important to note that while portfolio inflows are useful in bridging domestic savings and investment gap, they are also associated with volatilities in key macroeconomic variables such as monetary growth, inflation and exchange rate.

Iwueze, *et al.* (2013) observes that the growth or decline of a country's external reserves is an indispensable aspect of her economy. They then construct a statistical model that could be used to monitor the growth of external reserves in Nigeria necessary for economic policy formulation, implementation and monitoring. Specifically, the study (i) evaluated the external reserve data for the assumptions of autoregressive integrated moving average (ARIMA) model, (ii) determined the appropriate model for the study data and (iii) constructed a statistical model to forecast future external reserves situation in Nigeria.

The objective of this paper is to construct three statistical models of the external reserve data using the seasonal autoregressive integrated moving average (SARIMA), seasonal autoregressive integrated moving average with an exogenous input (SARIMA-X) and an autoregressive distributed lag (ARDL) process and evaluate the pseudo out-of-sample forecast performance of these models using some classical loss functions.

For ease of exposition, the paper is structured into five sections; with section one as the introduction. Section two discusses the methodological framework, while section three constructs the three proposed statistical models of external reserves. Section four elaborates on the pseudo out-of-sample forecast technique and section five concludes the paper.

2.0 Methodological Framework

2.1 The Seasonal ARIMA Process

Eni *et al* (2013) use the seasonal ARIMA process to study and model patterns of temperature in Warri, a town in Nigeria. Their chosen model is the SARIMA (1,1,1)(0,1,2)[12] process which met the criterion of model parsimony with low AIC value. The model was used to forecast temperature for 2009 and the forecast compared very well with the observed empirical data for 2009. Similarly Etuk (2012a, 2012b, 2012c, 2012d) uses the SARIMA process to model Nigeria's Gross Domestic Product, Consumer Price Index, inflation and Stock Prices and concluded that the models have been shown to be adequate. Ayinde and Abdulwahab (2013) identify a time series model forecast for crude oil exports in Nigeria for the period January 2002 to December 2011 through the use of SARIMA process. Based on the selection criteria used, SARIMA (1,1,1) (0,1,1) [12] was selected to be the best model to fit the Nigerian crude oil export data. Doguwa and Alade (2013) use SARIMA and SARIMAX processes to model Nigeria's headline, core and food inflation and provide 12 months forecast of the inflation types.

Following earlier studies, one of the methods of analysis adopted in this study is the Box and Jenkins (1976) and Box *et al.* (1994) procedure for fitting seasonal ARIMA model. Box *et al.* (1994) define the time series $\{y_t\}_{t\epsilon Z}$ as a seasonal ARIMA (p,d,q) (P,D,Q)[S] process if it satisfies the following equation:

$$\emptyset(L)\varphi(L^s)(1-L)^d(1-L^s)^D y_t = \theta(L)\Theta(L^s)\epsilon_t \qquad (1)$$

where L is the standard backward shift operator, φ and Θ are the seasonal autoregressive (AR) and moving average (MA) polynomials of order P and Q in variable L^s:

$$\varphi(L^s) = 1 - \varphi_1 L^s - \varphi_2 L^{2s} - \cdots - \varphi_P L^{Ps} \qquad (2)$$

$$\Theta(L^s) = 1 + \Theta_1 L^s + \Theta_2 L^{2s} + \ldots + \Theta_Q L^{Qs} \qquad (3)$$

The functions Ø and θ are the standard autoregressive (AR) and moving average (MA) polynomials of order p and q in variable L:

$$\phi(L) = 1 - \phi_1 L - \phi_2 L^2 - \cdots - \phi_p L^p \qquad (4)$$

$$\theta(L) = 1 + \theta_1 L + \theta_2 L^2 + \cdots + \theta_q L^q \qquad (5)$$

As an illustration, the SARIMA (1,1,1)(2,1,1)[12] model is a multiplicative model of the form:

$$(1-\phi_1 L)(1-\varphi_1 L^{12} - \varphi_2 L^{24})\Delta^s(\Delta y_t) = (1+\theta_1 L)(1+\Theta_1 L^{12})\epsilon_t \qquad (6)$$

Using the properties of operator L, it follows that equation (6) can be expressed as:

$$\Delta^s(\Delta y_t) = \phi_1\Delta^s(\Delta y_{t-1}) + \varphi_1\Delta^s(\Delta y_{t-12}) + \varphi_2\Delta^s(\Delta y_{t-24}) - \varphi_1\phi_1\Delta^s(\Delta y_{t-13}) - \varphi_2\phi_1\Delta^s(\Delta y_{t-25}) + \theta_1\epsilon_{t-1} + \Theta_1\epsilon_{t-12} + \theta_1\Theta_1\epsilon_{t-13} + \epsilon_t$$

where Δ^s is the seasonal differencing. Also, as an illustration, the ARIMA (1,1,1) model is of the form:

$$(1-\phi_1 L)\Delta y_t = (1+\theta_1 L)\epsilon_t \quad (7)$$

Using the properties of operator L, it follows that equation (7) can be expressed as:

$$\Delta y_t = \phi_1\Delta y_{t-1} + \theta_1\epsilon_{t-1} + \epsilon_t$$

where Δ is the difference operator. Also d and D are orders of integration and $\{\epsilon_t\}_{t\epsilon Z}$ is a Gausian white noise with zero mean and constant variance. Ideally S equals 12 for monthly data and 4 for quarterly data. For non-seasonal ARIMA, $\varphi(.)$ and $\Theta(.)$ assume a unit value as in equation (7). The details of ARIMA modeling procedure are contained in Box and Jenkins (1976), Box *et al.* (1994) and Asteriou and Hall (2007). For the external reserves series under study, the estimates of the parameters which meet the stationarity and invertibility conditions are obtained using the Eviews software.

The Box *et al.* (1994) procedure outlined above assumes that (i) the underlying distribution of the series under study is normal, (ii) the variance is constant and (iii) that the relationship between the seasonal and non – seasonal components is multiplicative. When one or all of these conditions are violated the fitted model may be inadequate for the series under study.

2.2 The Seasonal ARIMA-X Process

The SARIMA-X (or structural SARIMA) process differs from the SARIMA process ostensibly because it takes cognizance of an exogenous input, which consists of additional exogenous variables that could explain the behavior of the dependent variable. Thus, we define the time series $\{y_t\}_{t\epsilon Z}$ as a SARIMA-X (p,d,q) (P,D,Q)[S] process if it satisfies the following equation:

$$(1-L)^d(1-L^s)^D y_t = (1-L)^d(1-L^s)^D\psi' X_t) + \mu_t \quad (8)$$

$$\emptyset(L)\varphi(L^s)\mu_t = \theta(L)\Theta(L^s)\epsilon_t$$

The vector X_t constitutes other relevant exogenous variables that are difference stationary and ψ is the vector of parameter values. As an illustration, the seasonal ARIMAX (1,1,1) (2,1,1)[12] model with r exogenous and integrated variables $\{x_{it}, i=1,2\ldots r\}$ is a multiplicative model of the form:

$$\Delta^s(\Delta y_t) = c + \sum_{i=1}^{r} \gamma_i \Delta^s(\Delta x_{it}) + \mu_t \qquad (9)$$

with the autoregressive term $\{\mu_t\}_{t\epsilon Z}$ satisfying the following condition:

$$(1-\phi_1 L)(1-\varphi_1 L^{12} - \varphi_2 L^{24})\mu_t = (1+\theta_1 L)(1+\Theta_1 L^{12})\epsilon_t \qquad (10)$$

where c is a constant and $\{\gamma_k, k=1,2\ldots r\}$ are the parameters of the r exogenous variables used in the model and Δ^s is the seasonal difference operator. Using the properties of operator L, it follows that equation (9) can be expressed as:

$$\begin{aligned}\Delta^s(\Delta y_t) = {} & c + \sum_{i=1}^{r} \gamma_i \Delta^s(\Delta x_{it}) + \phi_1\mu_{t-1} + \varphi_1\mu_{t-12} + \varphi_2\mu_{t-24} \\ & - \varphi_1\phi_1\mu_{t-13} - \varphi_2\phi_1\mu_{t-25} + \theta_1\epsilon_{t-1} + \Theta_1\epsilon_{t-12} \\ & + \theta_1\Theta_1\epsilon_{t-13} + \epsilon_t\end{aligned}$$

where Δ, Δ^s and ϵ_t are as defined in equation (9). The exogenous variables considered for inclusion in the short-term forecasting models were exports (XP) and oil price (OP). All the variables are in millions of US dollars. We expect exports and international oil prices to be positively associated with external reserves (ER) because the larger the exports and the higher the oil prices the higher the likelihood of increased accretion to external reserves.

2.3 The ARDL Process

In the ARDL model proposed by Pesaran *et. al.* (2001), it is not necessary to ensure that all the included variables are integrated of order one as in Johansen cointegration framework. Using the bound testing approach, the ARDL (p,q,r) representation of the statistical model of external reserve ER is specified as:

$$dlog(ER_t) = \beta_o + \beta_1 \log(ER_{t-1}) + \beta_2 \log(XP_{t-1}) + \beta_3 \log(OP_{t-1}) + \sum_{i=1}^{p} \pi_i \text{dlog}(ER_{t-i}) + \sum_{j=1}^{q} \delta_j \text{dlog}(XP_{t-j}) + \sum_{m=1}^{r} \theta_m \text{dlog}(OP_{t-m}) + \in_t \quad (11)$$

where β_0 is a constant, β_1 to β_3 are the long-run parameters of the model, and π, $\boldsymbol{\delta}$ and θ are the short-run coefficients. The error term $\in_t$ is expected to be a white noise. The letters p, q and r are the optimal lag lengths that define the ARDL (p,q,r) model. The ARDL bound test for no cointegration among the variables against the presence of cointegration involves testing the null hypotheses of the absence of co-integration:

$$H_o: \beta_1 = \beta_2 = \beta_3 = 0 \quad Vs \quad H_1: \beta_1 \neq \beta_2 \neq \beta_3 \neq 0$$

The ARDL bound test is based on F Wald statistic and the asymptotic distribution of the statistic is non-standard under the null hypothesis of no co-integration. If the computed F Wald statistic lies above the upper bound critical value (or P-value of less than 10 per cent) the null hypothesis is rejected, indicating the existence of co-integration amongst the variables in the model:

$$\log(ER_t) = a_1 \log(XP_t) + a_2 \log(OP_t) + \mu_t \quad (12)$$

Once the presence of co-integration is established, an appropriate distributed lag error correction model of equation (12) is specified as follows:

$$dlog(ER_t) = \beta_o + \sum_{i=1}^{p} \pi_i \text{dlog}(ER_{t-i}) + \sum_{j=0}^{q} \delta_j \text{dlog}(XP_{t-j}) + \sum_{m=0}^{r} \theta_m \text{dlog}(OP_{t-m}) + \gamma \hat{\mu}_{t-1} + \in_t \quad (13)$$

3.0 Parsimonious Statistical Models of External Reserve

This paper models external reserves time series using monthly data of external reserve from January 2003 to May 2014 sourced from the CBN statistics database. The monthly trend of the external reserves data is illustrated in Fig. 1 in both actual and log transform version.

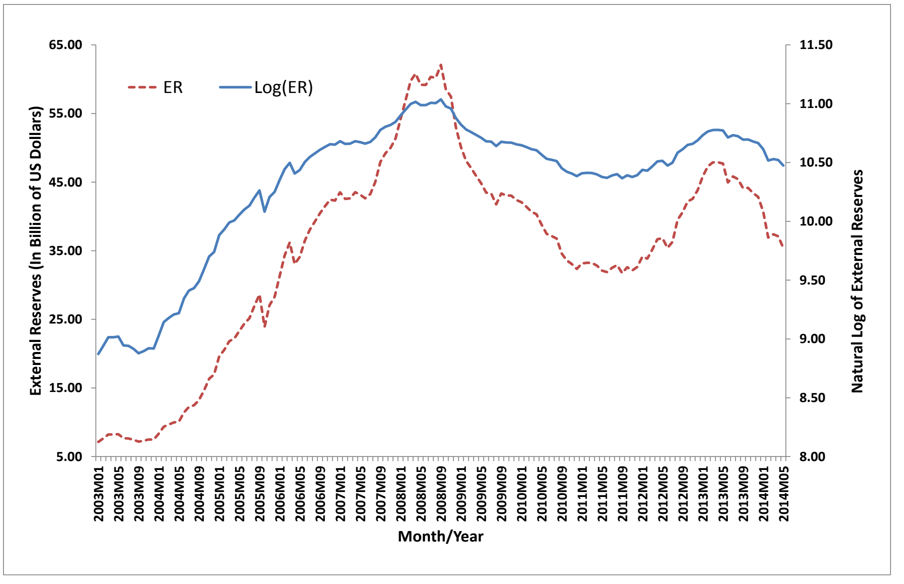


Fig. 1: Nigeria's External Reserves (January 2003 to May 2014)

The sample for the estimation and forecast evaluation spans the period January 2003 to May 2014 (137 observations) and is divided into two parts. The first part is the training sample, which includes all monthly data up to June 2013 (126 observations), and the second part is the forecasting sample, which includes the remaining data from July 2013 to May 2014 (11 observations). The paper uses the training sample to estimate the parameters of the forecasting models, while the forecasting sample is used for forecast evaluation.

Table 1: Testing the Null Hypothesis of a Unit Root Using ADF with intercept in the equation

Variable	ADF	P-Value	Variable	ADF	P-Value
$\log_e$(ER)	-3.5001	0.0095	dlog(ER)	-5.2553	0.0000
$\log_e$(XP)	-1.6103	0.4743	dlog(XP)	-14.9001	0.0000
$\log_e$(OP)	-1.7502	0.4037	dlog(OP)	-8.3581	0.0000

Using the Augmented Dickey Fuller (ADF) test for the null hypothesis of a unit root with intercept included on the test equation, all the variables presented in Table 1, except log(ER) suggest that the null hypothesis of a unit root cannot be rejected at the 1 per cent level. This suggest that log(XP) and log(OP) are not intercept stationary at their levels, but integrated of order one.

Therefore, their log-transforms would be used to fit the appropriate seasonal ARIMA, ARIMA-X and ARDL statistical models for the external reserves data.

To detect probable presence of trend, seasonality, time varying variance and other non-linear phenomena, the time plot of the external reserves data is examined side by side with the plots of sample autocorrelation functions (ACF) and sample partial autocorrelation function (PACF). This will assist us to determine the possible order of differencing and the need to employ logarithmic transformation to stabilize the variance. Non-stationary behavior is indicated by the refusal of both the ACF values ρ_k, and the PACF values φ_{kk}, to die out nippily. Also, possible seasonal differencing is indicated by large ACF values ρ_k at lags S, 2S,….,nS. Normally both simple and seasonal differencing are applied to the external reserves data until it becomes stationary - indicated by either a cut or exponential decay of ACF values as well as the PACF values.

Without loss of generality, the seasonal ARIMA model is difficult to identify by visual methods of the ACF and PACF plots only. These plots provide only a rough guess of possible values of p, q, P and Q from which several models shall be postulated and then use the model selection criterion of residual sum of squares RSS, Akaike's Information Criterion, AIC and Schwarz's Bayesian Criterion, SBC to choose the best model. The test for model adequacy requires residual analysis and is done by inspecting the ACF of the residual obtained by fitting the identified model. If the model is adequate then residuals should be a white noise process. Jarque-Bera normality test of the residuals is used to test the null hypothesis of normality, and rejection of the null hypothesis based on the significant p-value will lead to the conclusion that the distribution from which the residuals came is non-normal.

Before using these parsimonious models for statistical inference, the residuals ϵ_t are generally examined for evidence of serial correlation. The Breusch-Godfrey serial correlation LM test (BG LM F-statistic) is used to test the null hypothesis of no serial correlation up to a specific order in the residuals. Also to test the null hypothesis that there is no autoregressive conditional heteroskedasticity (ARCH) effect in the residuals, we employ the ARCH LM test. Accepting the null hypothesis will indicate that there is no ARCH effect in the residuals.

In this section we shall provide parsimonious models of external reserve directly using ARIMA, seasonal ARIMA, seasonal ARIMA-X and ARDL processes.

Table 2: Parameter Estimates of the Fitted ARIMA

Dependent Variable: dlog(ER_t)		
Estimated models	**ARIMA**	
Parameters	Estimate	P-Value
c	0.0127	0.0793
$\emptyset_1$	0.2329	0.0107
$\emptyset_2$	0.2111	0.0197
BG LM F-Statisitic	1.0106	0.3671
AIC	-3.3740	
SBC	-3.3054	
Jarque-Bera Normality Test	142.1980	0.0000
ARCH LM Test	9.0310	0.0032
Adjusted R-Squared	0.1136	

3.1 The Fitted ARIMA Model

Iwueze *et al.* (2013) fitted an ARIMA (2,1,0) to the logarithm-transformed monthly record of external reserves from January 1999 to December 2008. The ARIMA external reserve model fitted by Iwueze et al as ARIMA (2, 1, 0) is defined by

$$dlog(ER)_t = c + \mu_t$$

$$(1 - \emptyset_1 L^1 - \emptyset_2 L^2)\mu_t = \epsilon_t \quad (14)$$

Using the properties of operator L, it follows that equation (14) can be expressed as:

$$\mathrm{d}log(ER)_t = c + \phi_1\mu_{t-1} + \phi_2\mu_{t-2} + \epsilon_t$$

where ER, μ is the autoregressive term and ϵ is the moving average term or white noise. Log(.) is the natural log operator. Also the estimates of the parameters c, $\emptyset_1$ and $\emptyset_2$ are presented in Table 2.

3.2 The Fitted Seasonal ARIMA Model

We perform a logarithm and first regular difference on the monthly external reserves data to stabilize the variance and remove the trend. A time plot of the external reserves after logarithmic and first regular difference transformation is presented in Fig 2.

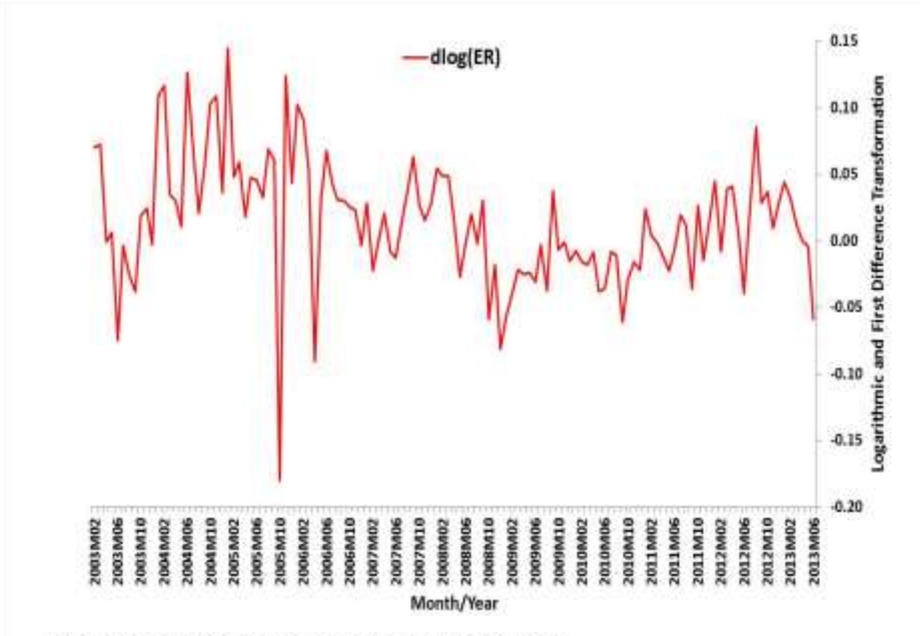


Fig 2: Time Plot of Transformed External Reserves Data

On examining Fig 2, we note the strong presence of seasonal factors and suspect the presence of seasonal trend. This is confirmed by the high spikes at and around the seasonal lags of the ACF as shown in Fig 3. Also, the ADF test indicates that both trend and intercept are significant, suggesting that the logarithmic and first difference transformed ER data is not trend and intercept stationary. We complete the ER data preparation process by performing a first order seasonal difference with the time plot shown in Fig 4.

Visual examination of Fig 4 indicates that the process is now stationary. The ADF test indicates that both trend and intercept are not significant suggesting that the ER data after logarithm, first regular difference and first seasonal difference transformations is now a stationary process of the external reserves. Thus we expect a seasonal ARIMA process of the form SARIMA (p,1,q)(P,1,Q)[12].

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.299	0.299	11.463	0.001
		2	0.275	0.203	21.208	0.000
		3	0.224	0.111	27.713	0.000
		4	0.237	0.124	35.107	0.000
		5	0.218	0.088	41.366	0.000
		6	0.319	0.203	54.942	0.000
		7	0.185	-0.003	59.545	0.000
		8	0.128	-0.051	61.760	0.000
		9	0.060	-0.086	62.252	0.000
		10	0.182	0.101	66.833	0.000
		11	0.124	0.009	68.990	0.000
		12	0.187	0.069	73.886	0.000
		13	0.100	-0.018	75.308	0.000
		14	0.174	0.107	79.618	0.000
		15	0.026	-0.084	79.714	0.000
		16	0.051	-0.071	80.097	0.000
		17	0.083	0.016	81.104	0.000
		18	0.085	-0.001	82.166	0.000
		19	0.178	0.173	86.913	0.000
		20	0.148	0.036	90.235	0.000
		21	0.068	-0.006	90.951	0.000
		22	0.110	0.039	92.819	0.000
		23	0.063	-0.031	93.442	0.000
		24	0.145	0.031	96.751	0.000
		25	0.145	0.021	100.09	0.000
		26	-0.005	-0.172	100.10	0.000
		27	-0.073	-0.130	100.96	0.000
		28	0.039	0.071	101.21	0.000
		29	-0.093	-0.143	102.63	0.000
		30	0.014	0.031	102.66	0.000
		31	-0.040	-0.080	102.93	0.000
		32	-0.033	0.035	103.12	0.000
		33	-0.035	0.057	103.33	0.000
		34	0.036	0.064	103.56	0.000
		35	-0.050	-0.037	103.99	0.000
		36	0.003	0.011	103.99	0.000

Fig 3: ACF and PACF Plot of dlog(ER_t)

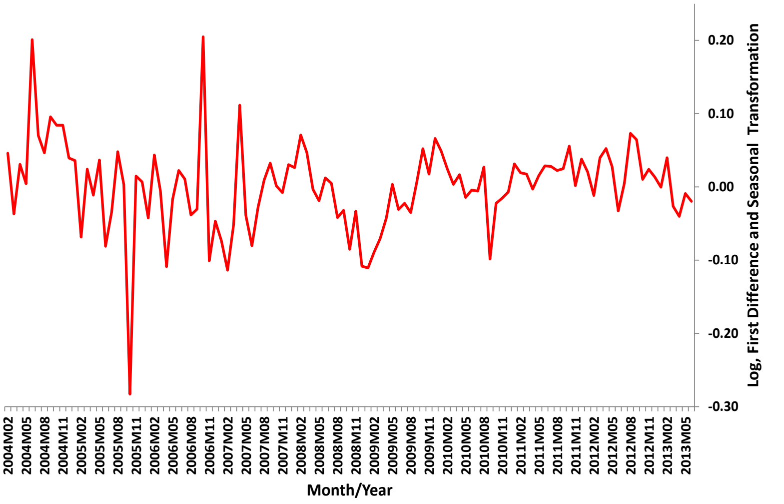
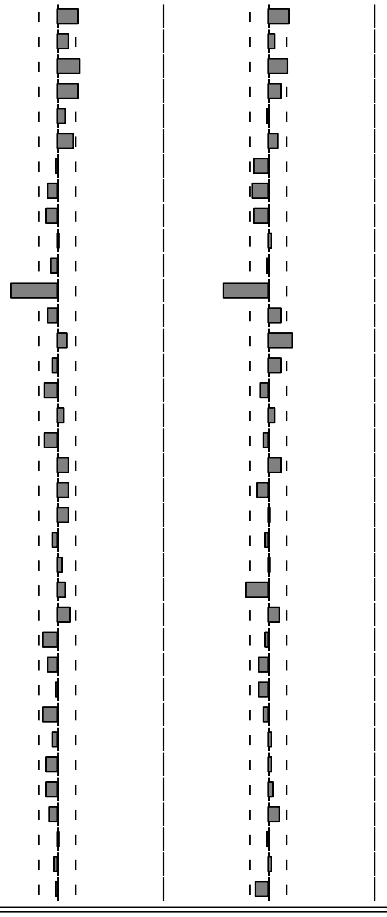


Fig 4: Time Plot of Transformed External Reserves Data (Δ^{12}dlog(ER_t))

Autocorrelation Partial Correlation



	AC	PAC	Q-Stat	Prob
1	0.218	0.218	5.4976	0.019
2	0.119	0.075	7.1527	0.028
3	0.230	0.201	13.423	0.004
4	0.217	0.139	19.022	0.001
5	0.085	-0.008	19.889	0.001
6	0.168	0.103	23.305	0.001
7	-0.000	-0.129	23.305	0.002
8	-0.090	-0.142	24.316	0.002
9	-0.101	-0.138	25.599	0.002
10	0.012	0.036	25.616	0.004
11	-0.062	-0.000	26.103	0.006
12	-0.444	-0.433	51.460	0.000
13	-0.094	0.132	52.601	0.000
14	0.090	0.249	53.676	0.000
15	-0.036	0.132	53.851	0.000
16	-0.119	-0.070	55.751	0.000
17	0.070	0.081	56.422	0.000
18	-0.114	-0.045	58.198	0.000
19	0.115	0.124	60.039	0.000
20	0.111	-0.106	61.767	0.000
21	0.118	0.003	63.722	0.000
22	-0.036	-0.024	63.905	0.000
23	0.062	0.010	64.460	0.000
24	0.087	-0.207	65.577	0.000
25	0.127	0.118	67.954	0.000
26	-0.128	-0.021	70.401	0.000
27	-0.094	-0.088	71.745	0.000
28	-0.014	-0.085	71.776	0.000
29	-0.137	-0.046	74.669	0.000
30	-0.048	0.033	75.024	0.000
31	-0.102	0.036	76.659	0.000
32	-0.106	0.059	78.450	0.000
33	-0.075	0.109	79.367	0.000
34	0.016	-0.001	79.409	0.000
35	-0.024	0.049	79.505	0.000
36	-0.012	-0.122	79.530	0.000

Fig 5: ACF and PACF Plot of $\Delta^{12}dlog(ER_t)$

The order of the model parameters p, q, P and Q are identified by visual inspection of the ACF and PACF of the stationary process of the external reserves presented in Fig 5 to propose possible models and the use of model selection criterion of AIC and SBC to select the most appropriate model. We could see that the ACF in Fig 5 cut at q=3 and Q=1, suggesting a moving average parameter of order 3 and a seasonal moving average parameter of order 1. Similarly from the PACF plot in Fig 5, we notice a cut at lag 3 and lag 24, suggesting an AR parameter of order three, p=3, and a seasonal AR parameter of order two, P=2. Based on these results, we postulate 14 models from which, based on the model selection criterion of residual sum of squares (RSS), AIC and SBC, the parsimonious model is selected. The postulated models together with selection criteria are presented in Table 3.

Table 3: Postulated Models and Performance Evaluation

Model	RSS	AIC	SBC	Normality Test		Serial Correlation		ARCH Effect	
				JB Test	P-Value	BG Test	P-Value	LM Test	P-Value
SARIMA(1,1,1)(2,1,1)[12]	0.0798	-4.031	-3.862	2.474	0.291	0.872	0.422	0.264	0.609
SARIMA(2,1,1)(2,1,1)[12]	0.0779	-4.019	-3.821	2.957	0.228	0.459	0.633	0.141	0.709
SARIMA(1,1,2)(2,1,1)[12]	0.0794	-4.013	-3.816	2.653	0.265	0.413	0.663	0.210	0.648
SARIMA(2,1,2)(2,1,1)[12]	0.0779	-3.996	-3.769	2.937	0.230	0.971	0.384	0.148	0.702
SARIMA(3,1,2)(2,1,1)[12]	0.0747	-4.001	-3.744	4.229	0.121	3.721	0.029	0.111	0.741
SARIMA(2,1,3)(2,1,1)[12]	0.0781	-3.971	-3.716	2.001	0.367	0.706	0.497	0.111	0.741
SARIMA(3,1,3)(2,1,1)[12]	0.0721	-4.014	-3.728	**7.296**	**0.026**	0.226	0.798	0.209	0.649
SARIMA(1,1,1)(1,1,1)[12]	0.1496	-3.567	-3.437	292.812	0.000	0.126	0.881	3.984	0.049
SARIMA(2,1,1)(1,1,1)[12]	0.1485	-3.543	-3.386	270.778	0.000	0.005	0.995	2.545	0.114
SARIMA(1,1,2)(1,1,1)[12]	0.1491	-3.551	-3.394	280.941	0.000	0.434	0.649	2.543	0.114
SARIMA(2,1,2)(1,1,1)[12]	0.1478	-3.528	-3.344	254.698	0.000	0.036	0.964	2.771	0.099
SARIMA(3,1,2)(1,1,1)[12]	0.1477	-3.497	-3.285	245.325	0.000	0.111	0.895	2.363	0.128
SARIMA(2,1,3)(1,1,1)[12]	0.1477	-3.508	-3.298	253.176	0.000	0.034	0.966	2.488	0.118
SARIMA(3,1,3)(1,1,1)[12]	0.1414	-3.519	-3.282	151.278	0.000	2.781	0.068	1.387	0.242

From Table 3, we note that in terms of AIC and SBC, the SARIMA (1,1,1)(2,1,1)[12] model performed best. However, it is in competition with SARIMA (3,1,3)(2,1,1)[12] that has the lowest RSS. Nonetheless, the residuals of this competing model are non-normal and cannot therefore be a white noise. The chosen seasonal ARIMA external reserve model is of the form:

$$\Delta^{12} dlog(ER_t) = c + \mu_t$$

$$(1 - \emptyset_1 L^1)\,(1 - \varphi_1 L^{12} - \varphi_2 L^{24})\mu_t = (1 + \theta_1 L^1)(1 + \Theta_1 L^{12})\epsilon_t \qquad (15)$$

Using the properties of operator L, it follows that equation (15) can be expressed as:

$$\Delta^{12}\text{dlog}(\text{ER}_t) = c + \phi_1\mu_{t-1} + \varphi_1\mu_{t-12} + \varphi_2\mu_{t-24} - \phi_1\varphi_1\mu_{t-13} - \phi_1\varphi_2\mu_{t-25} + \theta_1\epsilon_{t-1} + \Theta_1\epsilon_{t-12} + \theta_1\Theta_1\epsilon_{t-13} + \epsilon_t$$

where ER, μ and ϵ are as defined in equation (14). Also the estimates of the parameters c, $\emptyset_1$, φ_1, φ_2 θ_1 and Θ_1 are presented in Table 4.

We note that all the parameter values are significant at the 1 per cent level. To verify the suitability of the model, we plot the histogram and the ACF and PACF of the residuals in Figs 6 and 7. On inspection of Fig 7, there is no spike at any lag indicating that all the residual autocorrelations are not significantly different from zero. Moreover, the histogram of the residuals in Fig 6 shows that they are normally distributed with zero mean and constant variance indicating further the model adequacy.

Table 4: Parameter Estimates of the Fitted SARIMA

Dependent Variable: Δ^{12}dlog(ER_t)

Estimated model	**SARIMA**	
Parameters	Estimate	P-Value
c	0.0063	0.6539
$Ø_1$	0.1188	0.0000
φ_1	0.0797	0.0000
φ_2	0.0606	0.0000
θ_1	0.1805	0.0106
Θ_1	0.0254	0.0000
BG LM F-Statisitic	0.8720	0.4220
AIC	-4.0310	
SBC	-3.8621	
Jarque-Bera Normality Test	2.4740	0.2910
ARCH LM Test	0.2640	0.6090
Adjusted R-Squared	0.6268	

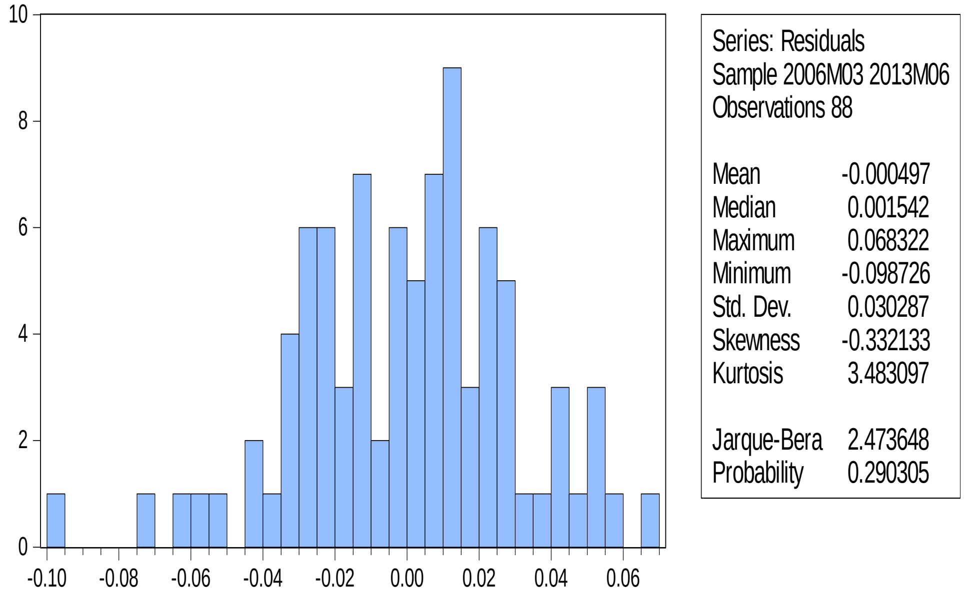


Fig 6: Histogram of Residuals of the Chosen Model

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.044	-0.044	0.1733	
		2	-0.007	-0.009	0.1779	
		3	-0.008	-0.009	0.1843	
		4	0.071	0.070	0.6534	
		5	0.124	0.130	2.1102	
		6	0.191	0.209	5.6191	0.018
		7	0.065	0.101	6.0380	0.049
		8	-0.110	-0.101	7.2401	0.065
		9	-0.057	-0.098	7.5669	0.109
		10	0.018	-0.052	7.6009	0.180
		11	-0.038	-0.121	7.7533	0.257
		12	-0.048	-0.121	7.9981	0.333
		13	-0.091	-0.114	8.8767	0.353
		14	0.077	0.131	9.5080	0.392
		15	-0.043	0.058	9.7048	0.467
		16	-0.185	-0.151	13.473	0.264
		17	-0.065	-0.051	13.942	0.304
		18	-0.088	-0.065	14.818	0.319
		19	0.138	0.157	17.014	0.255
		20	0.030	0.048	17.119	0.312
		21	0.126	0.190	19.006	0.268
		22	-0.104	0.061	20.293	0.260
		23	-0.070	-0.017	20.882	0.285
		24	-0.012	-0.125	20.899	0.342
		25	0.140	-0.029	23.358	0.272
		26	-0.015	-0.147	23.388	0.324
		27	-0.036	-0.126	23.558	0.371
		28	0.129	0.144	25.749	0.313
		29	-0.085	-0.011	26.717	0.318
		30	0.024	0.124	26.793	0.366
		31	-0.038	-0.050	26.996	0.410
		32	-0.019	-0.029	27.048	0.461
		33	0.032	-0.009	27.192	0.508
		34	0.089	0.058	28.361	0.499
		35	-0.090	-0.095	29.583	0.487
		36	-0.107	-0.050	31.335	0.449

Fig 7: ACF and PACF Plot of the Chosen SARIMA model

3.3 The Fitted Seasonal ARIMA-X Model

The stationary external reserve data is regress against the logarithm, first difference and seasonal difference (Δ^{12}) of the exogenous variables XP and OP to obtain:

$$\Delta^{12}\big(\mathrm{dlog}(\mathrm{ER}_t)\big) = \gamma_0 + \gamma_1\Delta^{12}\big(dlog(XP_{t-2})\big) + \gamma_4\Delta^{12}\big(dlog(OP_{t-3})\big) + \mu_t \qquad (16)$$

where the residual μ_t is expected to be modeled as a SARIMAX process. The OLS regression results of equation (16) are presented in Table 5. All the parameter estimates are correctly signed and significant. Though the residual is free from serial correlation, it deviates from the normality assumption. The augmented ADF test of the residual presented in Fig 8 indicates that the residual is now intercept and trend stationary, further confirming that the residual is a stationary process. Thus, we expect a seasonal ARIMA-X process of the form SARIMAX (p,1, q) (P,1,Q)[12].

Table 5: Parameter Estimates of the OLS

Dependent Variable: Δ^{12}dlog(ER_t)		
Estimated model	**OLS Regression**	
Parameters	Estimate	P-Value
γ_o	0.000284	0.9589
γ_1	0.060123	0.0119
γ_2	0.090888	0.0487
Adjusted R-Squared	0.1006	
AIC	-2.8427	
SBC	-2.7691	
Jarque-Bera Normality Test	132.2350	0.0000
BG LM F-Statistic	1.3051	0.2755
ARCH LM Test	0.0026	0.9596

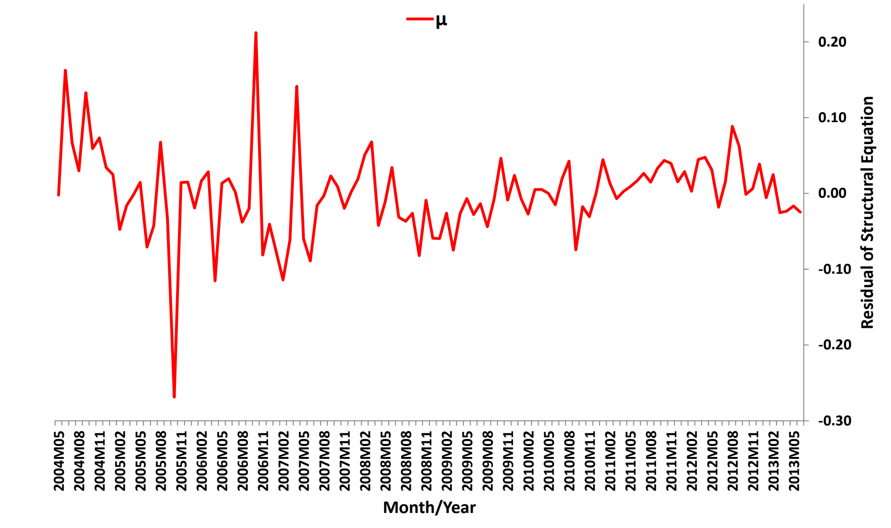
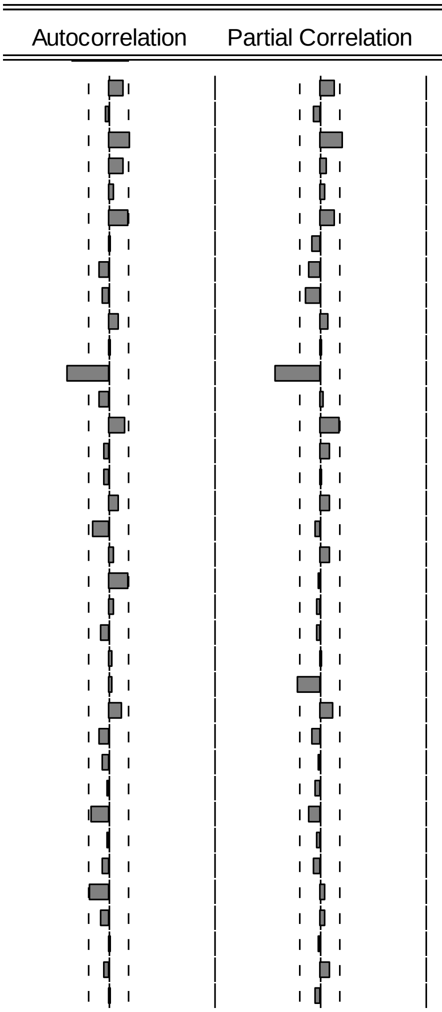


Fig 8: Time Plot of the Residual of Equation (16)



Lag	AC	PAC	Q-Stat	Prob*
1	0.146	0.146	2.3968	0.122
2	-0.027	-0.050	2.4821	0.289
3	0.220	0.237	8.0657	0.045
4	0.145	0.078	10.496	0.033
5	0.064	0.060	10.972	0.052
6	0.190	0.150	15.251	0.018
7	0.018	-0.076	15.289	0.032
8	-0.092	-0.106	16.317	0.038
9	-0.051	-0.124	16.628	0.055
10	0.100	0.086	17.861	0.057
11	0.024	0.022	17.933	0.083
12	-0.409	-0.430	38.933	0.000
13	-0.084	0.047	39.829	0.000
14	0.159	0.193	43.059	0.000
15	-0.042	0.105	43.286	0.000
16	-0.046	0.002	43.566	0.000
17	0.103	0.095	44.963	0.000
18	-0.155	-0.043	48.178	0.000
19	0.049	0.102	48.508	0.000
20	0.190	-0.012	53.449	0.000
21	0.053	-0.021	53.831	0.000
22	-0.075	-0.031	54.618	0.000
23	0.034	0.020	54.784	0.000
24	0.044	-0.213	55.056	0.000
25	0.132	0.124	57.573	0.000
26	-0.095	-0.071	58.893	0.000
27	-0.064	-0.001	59.506	0.000
28	-0.009	-0.038	59.517	0.000
29	-0.165	-0.104	63.666	0.000
30	-0.007	-0.021	63.675	0.000
31	-0.053	-0.064	64.117	0.000
32	-0.178	0.049	69.118	0.000
33	-0.067	0.053	69.842	0.000
34	0.024	-0.013	69.938	0.000
35	-0.040	0.098	70.201	0.000
36	0.006	-0.048	70.207	0.001

Fig 9: ACF and PACF Plot of μ of equation (16)

The order of the model parameters p, q, P and Q are identified by visual inspection of the ACF and PACF plot of μ presented in Fig 9 to propose possible models and the use of model selection criterion of AIC and SBC to select the most appropriate model. We could see that the ACF in Fig 9 cut at q=1 and Q=1, suggesting a moving average parameter of order 1 and a seasonal moving average parameter of order 1. Similarly from the PACF plot also in Fig 9, we notice a cut at lag 1 and lag 24 suggesting an AR parameter of order one, p=1, and a seasonal AR parameter of order two, P=2. Based on these results, we postulate 8 models from which, based on the model selection criterion of residual sum of squares RSS, AIC and SBC, the parsimonious model is selected. The postulated models together with the selection criteria are presented in Table 6.

Table 6: Postulated SARIMAX Models and Performance Evaluation

Model	RSS	AIC	SBC	Normality Test		Serial Correlation		ARCH Effect	
				JB Test	P-Value	BG Test	P-Value	LM Test	P-Value
SARIMAX(2,1,2)(2,1,1)[12]	0.0628	-4.122	-3.833	3.201	0.201	0.549	0.580	0.180	0.672
SARIMAX(1,1,1)(1,1,2)[12]	0.0843	-4.045	-3.833	66.055	0.000	3.158	0.047	1.809	0.294
SARIMAX(1,1,1)(1,1,1)[12]	0.1390	-3.566	-3.380	316.095	0.000	0.774	0.464	2.613	0.109
SARIMAX(2,1,1)(1,1,1)[12]	0.1368	-3.549	-3.335	286.577	0.000	0.711	0.494	1.794	0.183
SARIMAX(1,1,1)(2,1,1)[12]	0.0667	-4.124	-3.894	2.305	0.316	0.466	0.629	0.220	0.640
SARIMAX(2,1,2)(1,1,2)[12]	0.0782	-4.067	-3.799	4.774	0.092	2.105	0.128	1.927	0.168
SARIMAX(2,1,2)(1,1,1)[12]	0.1457	-3.465	-3.225	34.623	0.000	2.931	0.059	0.037	0.855
SARIMAX(1,1,2)(1,1,1)[12]	0.1385	-3.549	-3.337	311.004	0.000	0.942	0.394	1.234	0.269

From Table 6, we note that in terms of AIC and SBC, the SARIMAX (1,1,1)(2,1,1)[12] model performed best. However, it is in competition with SARIMAX (2,1,2)(2,1,1)[12] that has the lowest RSS. Nonetheless, the competing model is less parsimonious than the best chosen model. Therefore, our chosen seasonal ARIMAX external reserve model is of the form:

$$\Delta^{12} dlog(ER_t) = \gamma_o + \gamma_1 \Delta^{12} dlog(XP_{t-2}) + \gamma_2 \Delta^{12} dlog(OP_{t-3}) + \mu_t$$

$$(1 - \emptyset_1 L^1)\,(1 - \varphi_1 L^{12} - \varphi_2 L^{24})\mu_t = (1 + \theta_1 L^1)(1 + \Theta_1 L^{12})\epsilon_t \qquad (17)$$

Using the properties of operator L, it follows that equation (17) can be expressed as:

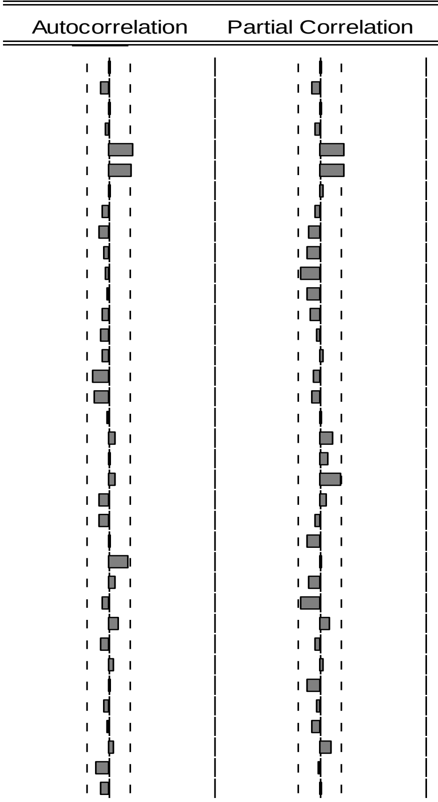
$$\Delta^{12}\mathrm{dlog}(\mathrm{ER}_t) = \gamma_o + \gamma_1 \Delta^{12} dlog(XP_{t-2}) + \gamma_2 \Delta^{12} dlog(OP_{t-3}) + \phi_1 \mu_{t-1} + \varphi_1 \mu_{t-12} + \varphi_2 \mu_{t-24} - \phi_1 \varphi_1 \mu_{t-13} - \phi_1 \varphi_2 \mu_{t-25} + \theta_1 \epsilon_{t-1} + \Theta_1 \epsilon_{t-12} + \theta_1 \Theta_1 \epsilon_{t-13} + \epsilon_t$$

where μ is the autoregressive term and ϵ is the moving average term or white noise. In addition, the estimates of the parameters γ_o , γ_1, γ_2, $\emptyset_1$, φ_1, φ_2 θ_1, and Θ_1 are presented in Table 6. We note that all the parameter values are significant except the constant. To verify the suitability of the model, we plot the histogram and the ACF and PACF of the residuals in Figs 10 and 11. On inspection of Fig 10, there is no spike at any lag indicating that all the residual autocorrelations are not significantly different from zero. Moreover, the histogram of the residuals in Fig 11 shows that the residuals are normally distributed with zero mean and constant variance.

Table 7: Parameter Estimates of the Fitted SARIMAX

Dependent Variable: $\Delta^{12}dlog(ER_t)$

Estimated model	**SARIMAX**	
Parameters	Estimate	P-Value
γ_o	-0.0030	0.6934
γ_1	0.0322	0.0306
γ_2	0.0665	0.0650
$\varnothing_1$	0.8442	0.0000
φ_1	-0.9115	0.0000
φ_2	-0.4228	0.0000
θ_1	-0.5522	0.0013
Θ_1	0.8789	0.0000
BG LM F-Statisitic	0.4660	0.6290
AIC	-4.1240	
SBC	-3.8940	
ARCH LM Test	0.2200	0.6470
Jarque-Bera Normality Test	2.3050	0.3160
Adjusted R-Squared	0.6621	



	AC	PAC	Q-Stat	Prob*
1	0.010	0.010	0.0086	
2	-0.067	-0.067	0.4076	
3	0.005	0.006	0.4097	
4	-0.028	-0.033	0.4824	
5	0.239	0.242	5.7750	
6	0.236	0.240	10.980	0.001
7	0.005	0.048	10.982	0.004
8	-0.059	-0.036	11.312	0.010
9	-0.098	-0.106	12.243	0.016
10	-0.043	-0.121	12.421	0.029
11	-0.030	-0.188	12.509	0.052
12	-0.009	-0.120	12.517	0.085
13	-0.064	-0.091	12.939	0.114
14	-0.082	-0.026	13.631	0.136
15	-0.054	0.036	13.941	0.176
16	-0.154	-0.065	16.493	0.124
17	-0.125	-0.075	18.202	0.110
18	-0.010	0.008	18.213	0.150
19	0.081	0.139	18.949	0.167
20	0.022	0.087	19.005	0.214
21	0.077	0.214	19.694	0.234
22	-0.088	0.067	20.611	0.244
23	-0.093	-0.036	21.641	0.248
24	0.027	-0.115	21.728	0.298
25	0.194	0.025	26.345	0.155
26	0.071	-0.104	26.973	0.172
27	-0.050	-0.188	27.292	0.200
28	0.095	0.101	28.455	0.199
29	-0.074	-0.039	29.184	0.213
30	0.059	0.037	29.646	0.238
31	0.001	-0.109	29.647	0.283
32	-0.034	-0.024	29.806	0.323
33	-0.002	-0.068	29.807	0.373
34	0.064	0.116	30.399	0.394
35	-0.110	-0.013	32.176	0.359
36	-0.067	0.033	32.847	0.377

Fig 10: ACF and PACF Plot of the Chosen SARIMAX model

the histogram and the ACF and PACF of the residuals in Figs 10 and 11. On inspection of Fig 10, there is no spike at any lag indicating that all the residual autocorrelations are not significantly different from zero. Moreover, the histogram of the residuals in Fig 11 shows that the residuals are normally distributed with zero mean and constant variance.

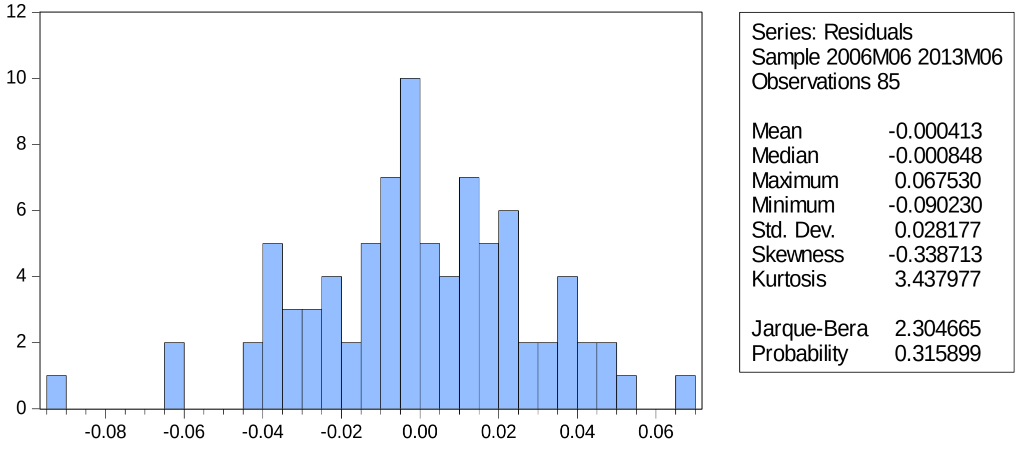


Fig 11: Histogram of Residuals of the Chosen SARIMAX Model

It is important to note that the coefficients of the exogenous variables satisfy the a priori expectations, since $\gamma_1 > 0$, $\gamma_2 > 0$. The parameter estimates of equation (17) and the diagnostics presented in Table 7 suggest the absence of ARCH effect and lack of autocorrelations in the residuals. Also, both the AR and MA terms are inverted suggesting that the fitted models are adequate for inference.

3.4 The Fitted ARDL Model

The unit root test conducted on the variables in equation (12) suggests the use of the ARDL approach to estimate the parameters of the model. Using monthly data from January 2003 to June 2013, the (estimated) orders of an ARDL(p,q,r) model in the three variables $\log(ER_t)$, $\log(XP_t)$ and $\log(OP_t)$ in equation (11) were selected by searching across the $6^3 = 216$ ARDL models, spanned by p = 1, 2, ….., 6, q = 1, 2,….,6 and r = 1, 2, ….,6, using the AIC criterion. This resulted in the choice of ARDL (6,1,5) specification for $d\log(ER_t)$ with estimates of the levels relationships given by

$$\log(ER)_t = 2.4161 + 0.9063\, Log(XP_t) + 0.0449\, Log(OP_t) + \hat{\pi}_t \qquad (18)$$

where $\hat{\pi}_t$ is the equilibrium correction term. The level estimates are highly significant except OP. The ARDL (6,1,5) bound test regression result is presented in Table 8, with the associated F Wald statistic and its p-value. The P-value for the computed F Wald statistic of the model is less than 10 per cent implying that the null hypothesis of no co-integration should be rejected. This suggests the existence of co-integration amongst the three variables.

Table 8: Regression Results of the ARDL (6,1,5) Bound Testing Approach

Dependent Variable: dlog(ER_t)		
Estimated models	**ARDL(6,1,5)**	
Variable	Coefficient	P-Value
C	0.3104	0.0184
log(ER_{t-1})	-0.0188	0.1256
log(XP_{t-1})	-0.0269	0.4184
log(OP_{t-1})	0.0280	0.4398
dlog(ER_{t-1})	0.1032	0.2630
dlog(XP_{t-1})	-0.0061	0.8486
dlog(OP_{t-1})	0.0970	0.0572
dlog(ER_{t-2})	0.0209	0.8200
dlog(OP_{t-2})	-0.0418	0.3853
dlog(ER_{t-3})	0.0768	0.3959
dlog(OP_{t-3})	0.0314	0.5200
dlog(ER_{t-4})	0.0700	0.4338
dlog(OP_{t-4})	-0.0192	0.6821
dlog(ER_{t-5})	0.0340	0.6977
dlog(OP_{t-5})	0.1307	0.0053
dlog(ER_{t-6})	0.2014	0.0209
F Wald Test	3.2047	0.0263
AIC	-3.4681	
SBC	-3.0944	
Adjusted R-Squared	0.2710	

The associated error correction model (ECM) regression associated with the level relationship in equation (18) is given in Table 9. The conditional ECM regression also passes the test against residual serial correlation as the hypothesis of no serial correlation is accepted in the model. The short term coefficients of the variables have the correct signs and significant for external reserves with six months lag and international oil price with one and five month lags. The ECM coefficient has the correct signed, but is not statistically significant.

Table 9: Equilibrium Correction Form of the ARDL (6,1,5) - the ARDL model of ER

Dependent Variable: dlog(ER_t)

Estimated models	**ARDL(6,1,5)**	
Variable	Coefficient	P-Value
C	0.0041	0.3512
dlog(ER_{t-1})	0.1449	0.1277
dlog(ER_{t-2})	0.0473	0.6132
dlog(ER_{t-3})	0.1075	0.2468
dlog(ER_{t-4})	0.0774	0.4007
dlog(ER_{t-5})	0.0383	0.6707
dlog(ER_{t-6})	0.2103	0.0185
dlog(XP_t)	0.0224	0.5027
dlog(XP_{t-1})	-0.0221	0.5164
dlog(OP_t)	0.0315	0.5768
dlog(OP_{t-1})	0.0903	0.0989
dlog(OP_{t-2})	-0.0496	0.3200
dlog(OP_{t-3})	0.0305	0.5430
dlog(OP_{t-4})	-0.0296	0.5392
dlog(OP_{t-5})	0.1098	0.0207
ECM_{t-1}	-0.0202	0.1157
BG-SC Test	0.2193	0.8035
AIC	-3.4135	
SBC	-3.0398	
Adjusted R-Squared	0.2301	

Our ARDL model of external reserves is, therefore chosen as:

$$dlog(ER_t) = \beta_o + \sum_{i=1}^{6} \pi_i \text{dlog}(ER_{t-i}) + \sum_{j=0}^{1} \delta_j \text{dlog}(XP_{t-j}) + \sum_{m=0}^{5} \theta_m \text{dlog}(OP_{t-m}) + \gamma\hat{\pi}_{t-1} + \epsilon_t \quad (19)$$

4.0 Performance Evaluations of the Fitted Models

A pseudo out-of-sample forecast technique, which is aimed at replicating the experience that a forecaster faces in a forecasting practice to evaluate the forecasting performance of a proposed model is used in the paper. The paper uses the training sample to estimate the parameters of the forecasting models and as a first step in our forecasting practice obtain one to six months ahead forecasts starting from July 2013 up to December 2013 from these models. The paper stores these forecasts by putting the first forecast (July 2013) as first entry in the series 1 step ahead, the second forecast (August 2013) as the first entry in the series 2 steps ahead and so on to the sixth forecast (December 2013) as the first entry in the series 6 steps ahead.

The actual data for July 2013 is added to the training sample after which the parameters of the models are re-estimated. Using the re-estimated models, we forecast the values from August 2013 up to January 2014. We then store these forecasts by putting the first forecast (August 2013) as the second entry in the series 1 step ahead, the second forecast (September 2013) as the second entry in the series 2 steps ahead and so on to the sixth forecast (January 2014) as the second entry in the series 6 steps ahead.

The above exercise is performed repeatedly until we reach the end of the pseudo out-of-sample period (May 2014). In this way, each of the forecast exercise yielded 6 series obtained as forecasts from one month ahead to 6 months ahead, which are then stored accordingly. A series of 6 observations was generated for each time horizon. We then tested the quality of the obtained forecasts using three classical statistical loss functions: Mean Absolute Error (MAE), Mean Absolute Percent Error (MAPE) and Root Mean Squared Error (RMSE), defined as follows. Let the series $y_{1t}, y_{2t},..., y_{6t}$ be the natural log of the actual external reserve numbers and $\hat{y}_{1t}, \hat{y}_{2t}, \ldots, \hat{y}_{6t}$ be the forecast values for the forecast horizon t = 1, 2, 3,….6, then:

$$MAE_t = \frac{1}{6}\sum_{i=1}^{6}|y_{it} - \hat{y}_{it}| \tag{20}$$

$$MAPE_t = \frac{1}{6}\sum_{i=1}^{6}\frac{|100(y_{it} - \hat{y}_{it})|}{y_{it}} \tag{21}$$

$$RMSE_t = \sqrt{\frac{1}{6}\left\{\sum_{i=1}^{6}(y_{it} - \hat{y}_{it})^2\right\}} \qquad (22)$$

The two scaled-dependent statistical loss functions: MAE_t and $RMSE_t$ and the scaled-independent measure $MAPE_t$ for the t forecast horizon (t=1, 2 .., 6) are used to compare the forecast performances of the estimated short-term forecasting models.

Table 10: Statistical Loss Functions for External Reserves

Forecast Horizon	MAE ARIMA	MAE SARIMA	MAE SARIMAX	MAE ARDL	MAPE ARIMA	MAPE SARIMA	MAPE SARIMAX	MAPE ARDL
1	824.7	901.3	1,078.1	511.2	1.8369	1.9899	2.3796	1.1330
2	1,794.8	1,609.3	2,094.1	1,300.3	3.9966	3.5747	4.6522	2.8991
3	3,200.7	2,480.9	2,994.2	2,689.0	7.0978	5.5071	6.6368	5.9831
4	4,764.0	3,081.0	3,343.8	3,923.6	10.4481	6.7591	7.3236	8.6367
5	6,446.7	3,050.7	3,125.3	5,211.2	13.9756	6.6751	6.8153	11.3318
6	8,326.1	3,839.6	3,862.9	6,742.5	17.7746	8.2274	8.2408	14.4166

We examine the four parsimonious short term forecasting models for external reserve with the view to assessing their pseudo out-of-sample forecast accuracy. The statistical loss functions employed for this purpose are the mean absolute error (MAE), the root mean squared error (RMSE) and the mean absolute percentage error (MAPE). The performance evaluation of the competing models is to determine which of them is more precise and reliable for forecasting external reserve over the six months forecast horizon.

We compute MAE, MAPE and RMSE defined in equations (20), (21) and (22) for the external reserve chosen models (14), (15), (17) and (19) using the performance evaluation framework described in section three. The results of the computations are presented in Table 9 and Fig 12.

All the three performance evaluation measures presented in Table 10 and Fig 12 provide similar results for external reserves. The fitted ARIMA model of Iwueze, *et al* (2013) consistently provides the largest forecast error in the three to six month forecast horizon. This suggests that the chosen models based on the ARDL, seasonal ARIMA and seasonal ARIMAX processes would be better in forecasting three to six month Nigeria's external reserves. For one to two months forecast horizon, the ARDL chosen model outperforms the other models and should be used to forecast external reserves in these horizons. The chosen SARIMA model consistently provides the smallest forecast error in the three to six month horizon, indicating that the seasonal ARIMA chosen model of external reserves is better in making three to six months ahead forecast.

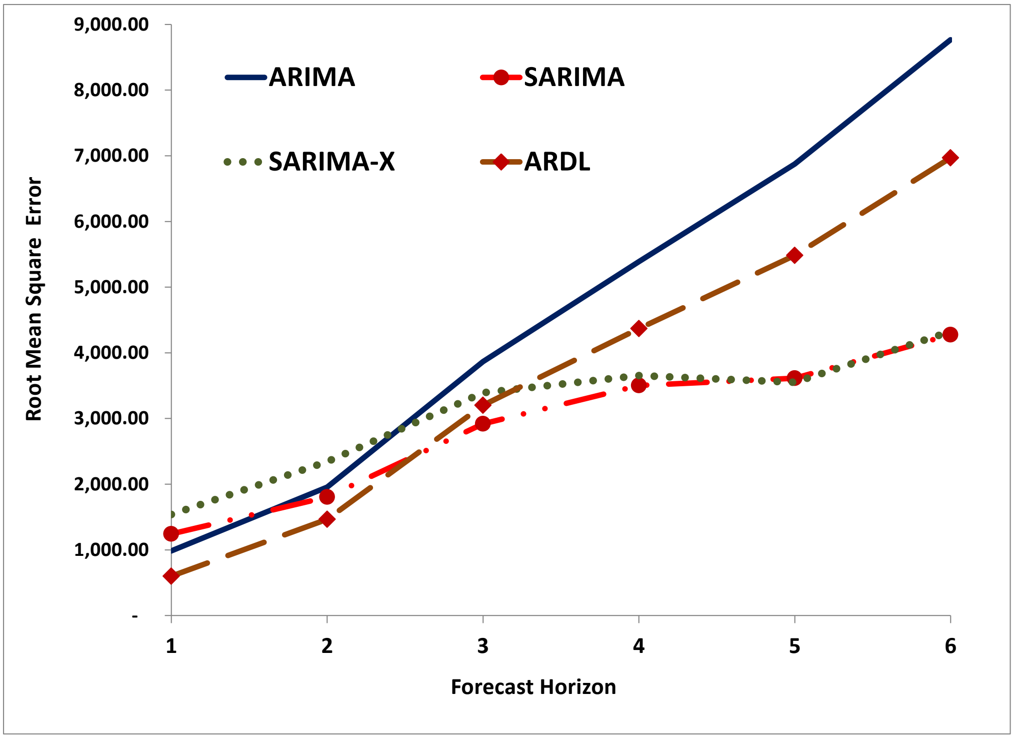


Fig 12: RMSE of the Chosen External Reserves models

5.0 Summary and Conclusions

This paper proposed three statistical models for external reserve data using both the ARDL, seasonal ARIMA and the seasonal ARIMA-X processes and evaluated the pseudo out-of-sample forecast performance of the models using three statistical loss functions - mean absolute error, mean absolute percent error and root mean squared error.

The results indicated that the ARDL outperforms the ARIMA, the seasonal ARIMA and the seasonal ARIMAX chosen models for one to two months forecast horizon. In contrast, the seasonal ARIMA chosen model appeared to have the smallest forecast error in the three to six months forecast horizon than the other models. In conclusion, we suggest that the ARDL should be used to provide one to two months ahead forecast of Nigeria's external reserves. However, for a longer forecast horizon of three months and above, the seasonal ARIMA model should be used. In forecasting external reserves at higher horizon, the paper concludes that seasonality is important and the seasonal ARIMA should be used by the Bank.

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